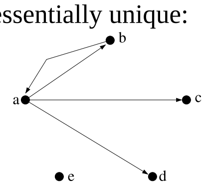
Discrete Mathematics: Tutorial 22 Solutions

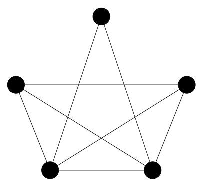
1. *Draw a directed graph with 5 vertices having the following in and out degrees:*

	a	b	c	d	e
in-degree	1	1	1	1	0
out-degree	3	1	0	0	0

   Answer: The structure is essentially unique:

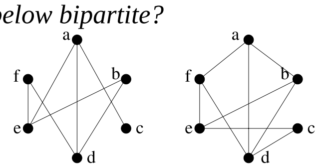


2. *Draw a simple graph with five vertices, and degree sequence 2,3,3,4,4.* **Answer**: The arrangement of edges and vertices is of course arbitrary, but the structure is essentially fixed since two of the vertices must be adjacent to all of the other four, and that just leaves one edge to add to make two of the other vertices of degree 3.



3. *Can a graph have degree sequence 3,4,4,5,6,6,6,6,7? Can a simple graph have degree sequence 3,3,3,3,5,5?* **Answer**: For the first part no, as the sum of this sequence is odd. For the second part yes. Take six vertices. Connect two of them to all the others. These two have degree 5 at this point. Divide the remaining four vertices into two pairs, and draw an edge connecting the vertices in each pair.

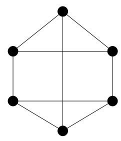
4. *Are the two graphs shown below bipartite?*



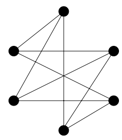
Graph A Graph B

   Answer: The first one is, we can take as the two parts: $\{a, b, f\}$ and $\{c, d, e\}$, that is, the top half and bottom half of the vertices. All the edges go between the top half and the bottom half, so that's enough. The second one is not bipartite. Whatever part d is in, a, b, c and f (its neighbours) would have to be in the other part. But a and f are joined by an edge, so the graph is not bipartite.

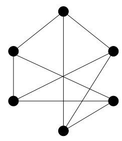
5. *Two of the three graphs shown below are isomorphic. Which are they? Why are they not isomorphic to the third?*



Graph A



Graph B



Graph C

Answer: Graphs A and C are isomorphic. They both consist of two triangles with corresponding vertices joined up in pairs. Despite the fact that their degree sequences are the same, they are not isomorphic to Graph B. For instance A and C contain triangles, while B does not (B is in fact the complete bipartite graph $K_{3,3}$.)